

Toward a Solution of the N-person Prisoner's Dilemma  
and  
An Application to Explain the Evolution of States

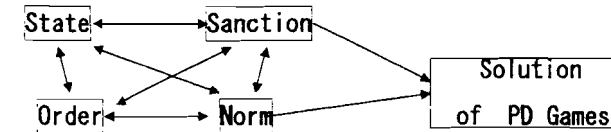
Hideki Kamiyama  
(Japan Society for the Promotion of Science)  
X V World Congress of Sociology  
11/7/2002  
rxg00156@nifty.ne.jp

CONTENTS

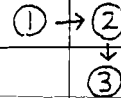
1. Introduction  
Overview, Answer, NPD  
and Evolutionary Game.
2. Model & Theorem  
Payoff of War, Inter-Group  
Evolutionary Game, "all  
D is not ESS" .
3. Theory

• We Can Interpret Basic Sociological Questions such as Formation  
of States As Solution of Prisoner' s Dilemma Game.

(Hobbes:1651=1965), (Parsons:1937), (Olson:1965), (Runciman and Sen:1965), (Hardin:1968),  
(Hardin:1971), (Ulman-Margalit:1977), (Axelrod:1984), (Taylor:1987), (Coleman:1990) ...



Two Person PD – N-person' s PD  
Game with an NE – Evolutionary Game with an ESS  
(Evolutionarily Stable Strategy)



• QUESTION

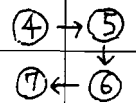
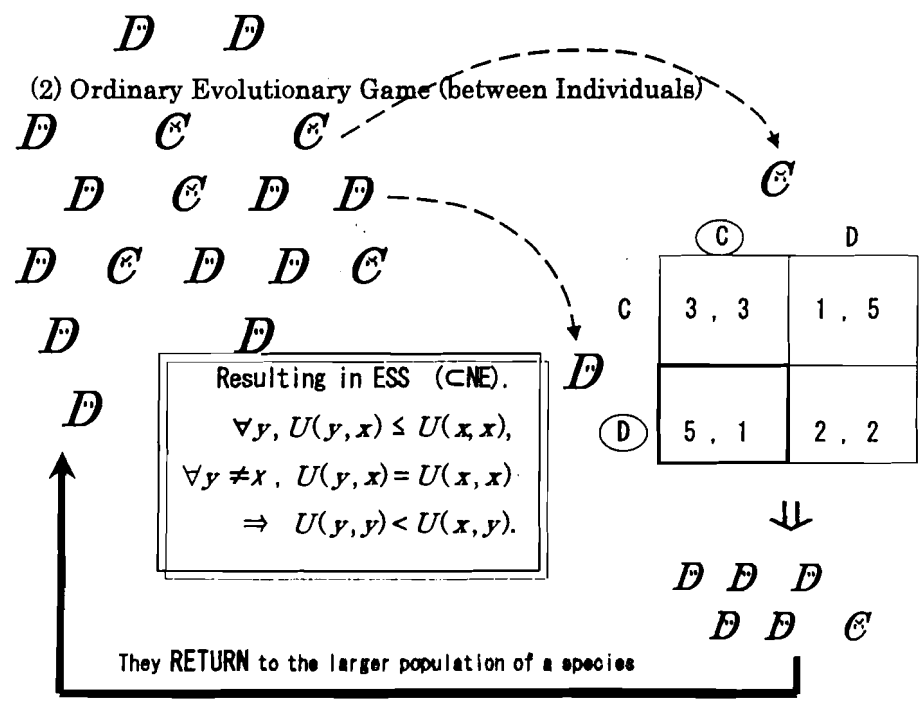
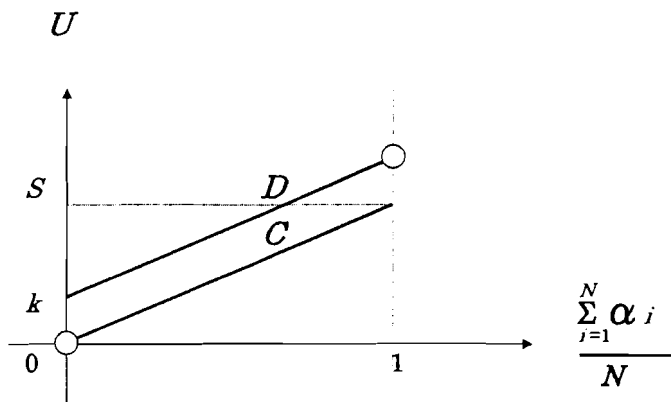
Although We Ought to Choose "D" as Rational Choices in NPD  
Situations, States usually have Existed in History. ... WHY ?

• ANSWER

... Wars between Groups(Herds) usually Existed in Evolution,  
Consequently, We Got Tendencies to Choose "C" as Biological  
Drive.

(1) N-person PD

Each Individual Strategy  $(\alpha_i^C, 1 - \alpha_i^D)$ ,

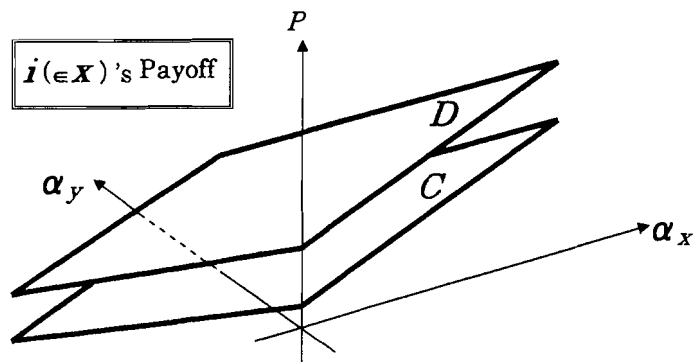


For Example, When the Interaction is WAR . . .

Group  $X$  and Group  $Y$  (each group has  $n$  Individuals) Exist.

$$\alpha_x = \sum_{i \in X} \alpha_i / n, \text{ when strategy of } i \in X \text{ is } (\alpha_i, 1 - \alpha_i).$$

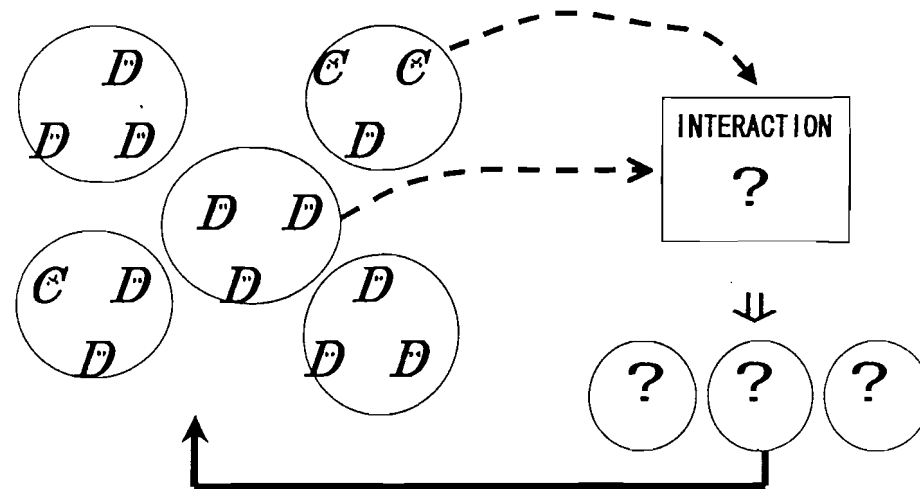
$$\alpha_y = \sum_{j \in Y} \alpha_j / n, \text{ when strategy of } j \in Y \text{ is } (\alpha_j, 1 - \alpha_j).$$



... Of course, When the Game is PD, an ESS is  $D$ .

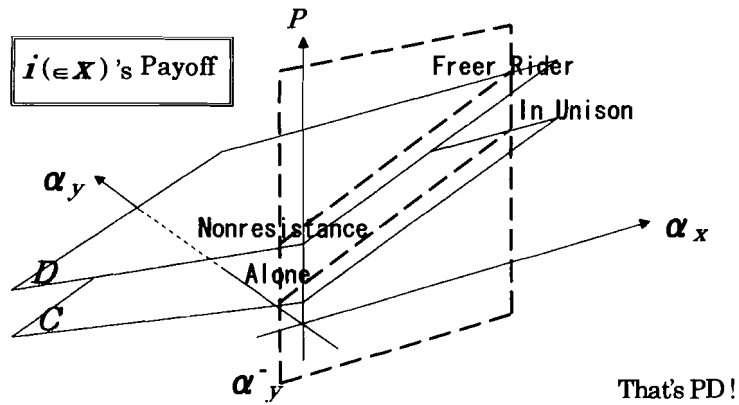
But, Groups (Herds) Always Existed in Reality!

The Model (Toward INTER-GROUP Evolutionary Game Models)



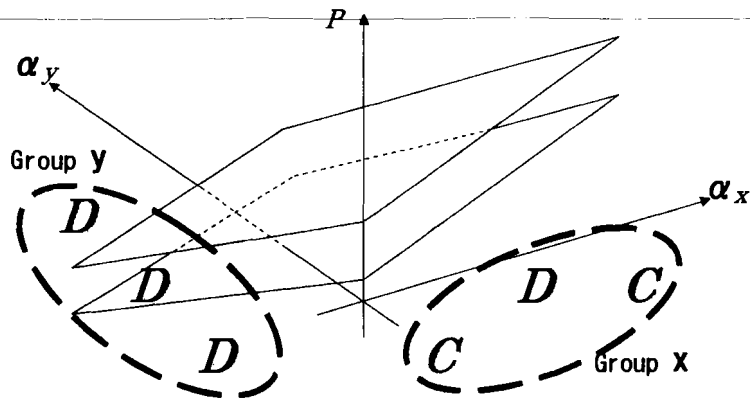
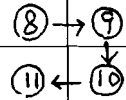
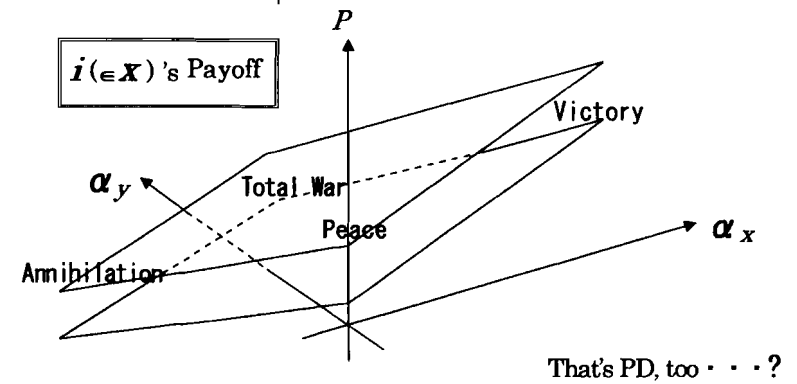
**GAME IN A GROUP**

		$i \in X$	
		Warlike Behavior $C$	Pacifist Behavior $D$
The Others $\in X$	W.B. $C_s$	Fighting in Unison	< Free Rider
	P.B. $D_s$	Fighting Alone	< Nonresistance



**GAME BETWEEN GROUPS ( ? )**

		Group $x$	
		W.B. <i>all</i> $C_s$	P.B. <i>all</i> $D_s$
Group $y$	W.B. <i>all</i> $C_s$	Total War	> Annihilation
	P.B. <i>all</i> $D_s$	Emphatic Victory	> Peace



⇓

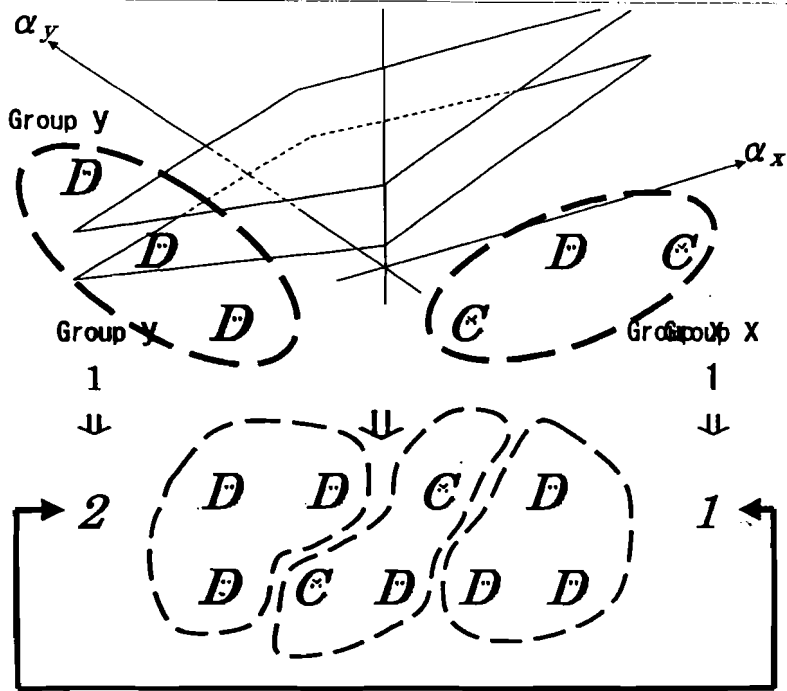
D D C D  
D C D D D

The NE of the Game in A Group is *all D*, the NE of the "Game" between Groups is "*all C*". If So, One of Those Will be Solved!

But, Beforehand, . . .

Can Groups Play Games REALISTICALLY ?

(Yes. If it is Evolutionary Game.)



Fitness of Groups, That is, PAYOFF of Groups.

(12) → (13)  
↓  
(14)

Payoffs of Groups Exist Uniquely on the Assumption that . . .

1. Groups Must be Formed.
2. The Number of Individuals in Each Group is the Same.
3. Only One Mutant Group Can Intrude on a Species At Any Period (as in the case of ordinary evolutionary game).

(For Details, See Appendix.)

THEOREM

If the Interaction of Groups is War, Group Strategy "all D" is not ESS of the Inter-Group Evolutionary Game.

PROOF

See Appendix

Theory

1. Repeated Wars between Groups on the Process of Evolution Solved NPD. Various Public Goods Could be Supplied to Strengthen the Group. Then a State That Plays Those Special Functions was Formed.
2. Probably, We have "the Residue" of the Evolution Process as Our Cognitive Frame and Form of Emotion. So, I Believe That NPDs can be Solved When People Find Groups.

### Note

(p.2) For example, we can consider as follows: A question "why the social order can be existed" is the basic matter on sociology. Since order is considered to be one of the public goods, we can never find the answer unless we solve NPD. Some may say that norm or sanction maintains order. Though, NPD should be solved to explain the occurrence of norm. Also, state mainly gives sanction, though we still have to solve NPD since cost payer is necessary to maintain the state.

(p.2) I will not focus on the two persons prisoner's dilemma on this report. Because one of the solution on two persons prisoner's dilemma has been indicated, introducing "repetition".

(p.6) This model is influenced by "group selection model." Basic idea of group selection model can be expressed as the following: several villages with common are supposed to be there. A village with people who restrain to set cows free would last longer. Contrary, one, which doesn't do it, would last earlier. Possibility of NPD solution is found in the difference of possibility of village "selection." In fact, such idea has existed in biology for a long time. Ooura of Japan widely introduced the research of Wilson which is the modern version of it. Also, original group selection model was presented, connecting so-called Hobus Issue on sociology and this idea. (Wilson: 1975),(Ooura : 2002)

Inter-group evolutionary game model succeeds to group selection model at the unique point to reinterpret the classic issue on sociology as human evolution issue and at the point to focus on groups then. On the other hand, there are some primary differences. Firstly, how to define a group in each model: group is the place to make frequent interaction between individuals on group selection model. On inter-group evolutionary game model, group itself is a unit of interaction. Secondly, formation of groups: primary idea regarding to NPD solution by group selection model is mentioned above. Though, if only this idea is available, we can rarely solve NPD or it is just for a while. Even if groups with people who choose C temporary gain, individuals who choose D sooner or later gain rapidly. Therefore, recent group selection model is created with some special settings about production, extinction or reorganization of groups. On inter-group evolutionary game model, group formation is to be given.

Additionally, I consider that lost idea of final solution, such as Nash equilibrium or ESS, brought about main theoretical difficulty of group selection model. Prediction needs the application of general solution of the idea. Otherwise, models can be haphazard.

(p.8.9) This figure is considered to be not only the war of our ancestors a long time ago. It precisely shows the essence of war that has been repeated until today, or our complicated dilemma we have about war.

(p.11) Inter-Group Game needs the assumption that there are sufficient number of individuals in the groups, but it's just for explanation.

(p.13) "The number of individuals in each group is almost the same" . Though, they are unlikely to be exactly, the size of the groups should be similar. "At any period in the process of evolution, even if there was another group strategy to the dominant group strategy, each group should have only one strategy" . In other words, no more than two group strategies can exist in the same period. It is the same assumption as normal evolutionary game between individuals. ESS needs to be considered if such a strategy is dominant for a long time.

(p.13) Additionally, the following assumption will be needed: each individual can judge if those around it belong to its own group. Even chimpanzees and apes can distinguish individuals.

(p.15) Additionally, third, in case to consider nation, order, rules or sanction in the modern society, this research indicates to take enough mention to the other society which the society has further relation but not only observing inside of the society. Inside base of a society is considered to be decided and be changed by the relation with other society. Of course, it coincides our regular observance. I would expect that the reported model could be useful in such researches.

## Appendix: The Solution of the NPD using an Inter-Group Evolutionary

### Game Model

#### 1. The definition of an inter-group game

A finite game in normal form may be summarized as a triplet  $G=(I,\theta,u)$ , where  $I$  is its player set,  $\theta$  its mixed-strategy space, and  $u$  its combined mixed-strategy payoff function. Let  $I=\{1, 2, \dots, i, \dots, 2n\} (n \geq 2)$  be divided into two parts, and let  $I^{(n)} = \{L, M\} = \{\{1, 2, \dots, i, \dots, n\}, \{n+1, n+2, \dots, m, \dots, 2n\}\}$  be the set of the two parts. When  $x=(x_1, x_2, \dots, x_i, \dots, x_m, \dots, x_{2n})$  is a strategy profile in game  $G$ , let  $x_L=(x_1, \dots, x_i, \dots, x_n)$  and  $x_M=(x_{n+1}, \dots, x_m, \dots, x_{2n})$  be the strategy profiles of players belonging to  $L$  and  $M$ , respectively. Let us assume that player  $i$  is an element of  $L$ . Then,  $L$  and  $M$  are groups of players in game  $G$ , if

$$\forall i \in L, \forall x, y \in \theta, \sum_{l=1}^n x_l = \sum_{l=1}^n y_l \text{ and } \sum_{m=n+1}^{2n} x_m = \sum_{m=n+1}^{2n} y_m \Rightarrow u_i(x_i, x_{-i}, x_m) = u_i(y_i, y_{-i}, y_m) \quad (1.1)$$

(if  $n$  is sufficiently big,  $\sum_{l=1}^{i-1} x_l + \sum_{l=i+1}^n x_l \doteq \sum_{l=1}^n x_l$  and  $u_i(x_i, x_{-i}, x_m) \doteq u_i(x_i, x_i, x_m)$ )

When profile  $x$  contains  $K$  kinds of strategies, *i.e.*, when all the strategies comprising  $x$  are elements of the set  $\{x_1, \dots, x_k, \dots, x_K\}$ , let  $n_{l, x_k} = \#\{i \in L | x_i = x_k\}$  denote the number of players in group  $L$  choosing strategy  $x_k$ , and let  $v_{l, x_k} = n_{l, x_k} / n$  denote the share of individual strategies  $x_k$  in group  $L$ .

$$x_l^{(n)} = \sum_{k=1}^K v_{l, x_k} \cdot x_k$$

can be called a *group strategy* of group  $L$ . Let  $\theta^{(n)}$  be the set of group strategy profiles  $x^{(n)} = (x_l^{(n)}, x_m^{(n)})$

If  $K=2$  and  $n_{l,x1} \cdot n_{m,x2} - n_{m,x1} \cdot n_{l,x2} \neq 0$  or  $n_{l,x2} = n_{m,x2} = 0$ , then

$$(u_l^{(n)}(x^{(n)}), u_m^{(n)}(x^{(n)})) \begin{pmatrix} n_{l,x1} & n_{l,x2} \\ n_{m,x1} & n_{m,x2} \end{pmatrix} = (n_{l,x1} \cdot u_{i \in I}(x_{i,1}, x_{l-i}^{(n)}, x_m^{(n)}) + n_{m,x1} \cdot u_{j \in m}(x_{j,1}, x_l^{(n)}, x_{m-j}^{(n)}) \\ , n_{l,x2} \cdot u_{i \in I}(x_{i,2}, x_{l-i}^{(n)}, x_m^{(n)}) + n_{m,x2} \cdot u_{j \in m}(x_{j,2}, x_l^{(n)}, x_{m-j}^{(n)})) \quad (1.2)$$

(1.2) defines the functions  $u_l^{(n)}: \theta^{(n)} \rightarrow \mathbb{R}$  and  $u_m^{(n)}: \theta^{(n)} \rightarrow \mathbb{R}$ . In addition, the combined function  $u^{(n)}$  is defined as  $u^{(n)} = (u_l^{(n)}, u_m^{(n)})$ . Therefore, the inter-group game can be expressed as  $G^{(n)}(I^{(n)}, \theta^{(n)}, u^{(n)})$ .

It is natural that  $u^{(n)}$  is regarded as the payoff of the groups, when we consider the inter-group game as an evolutionary game where the groups are constituted as if each group was a player. This is because the payoff is considered as the expected number of surviving offspring of a player who adopts a strategy in an evolutionary game. If it is possible to assume that the number of individuals that constitute a group is constant in any of the groups,  $u^{(n)}$  means the magnitude of potentiality, which is the likelihood that the group using each group strategy is reproduced after the groups have played the game. This definition of a payoff is particularly useful when we want to know whether a specific strategy is evolutionarily stable.

## 2. An ESS as the solution concept of an inter-group evolutionary game

Let us assume that some real situation can be represented in a model as an inter-group game. In such a case, under what conditions can the result of the situation be predicted by using the concept of an ESS? That is, what facts lie behind such a situation or what kind of facts motivate an ESS in an inter-group game?

As is already known, an ESS is used to predict the outcome of an ordinary evolutionary game between individuals. Its use is based on the assumption that a random interaction really



exists between a number of individuals. In accordance with this assumption, we must presume that there are a number of groups present and that random interactions occur between the groups in order to motivate the use of ESS in the inter-group game. Specifically, when we detect an inter-group game, it must be regarded as occurring as one of many interactions in that situation.

In the ordinary evolutionary game between individuals, “selection” is deeply related to the concept of an ESS. In the following, we discuss this further. In an evolutionary game between individuals, strategy  $x$  is evolutionarily stable if and only if

$$\forall y, u(x,x) \geq u(y,x), \quad (2.1)$$

$$\forall y \neq x, u(x,x) = u(y,x) \Rightarrow u(x,y) > u(y,y). \quad (2.2)$$

These are equivalent under the following condition:

$$u[x, \varepsilon y + (1-\varepsilon)x] > u[y, \varepsilon y + (1-\varepsilon)x], \quad (2.3)$$

where  $\varepsilon$  is the population share of a small population that uses a mutant strategy on entering a large population of individuals using incumbent strategy  $x$ . Inequality (2.3) expresses the biological intuition that evolutionary forces select against the mutant strategy if and only if the post-entry payoff (fitness) obtained from the mutant strategy is smaller than that of the incumbent strategy (Weibull, 1995). Therefore, inequality (2.3) motivates the use of conditions (2.1) and (2.2) as equivalent to this to predict the solution of the game. In the inter-group game, what conditions motivate the use of an ESS? In (2.3), what corresponds to  $\varepsilon$ , the fraction of the group that plays a group strategy different from the incumbent group strategy (we will call this a *mutant group* hereinafter). Let  $N$  be the total number of individuals in a species and let  $v_l$  be the share of mutant individuals in group  $L$ . Then,

$$\varepsilon N = \sum_{l=1}^N (v_l n). \quad (2.4)$$

We try to relate the number of mutant individuals to the number of mutant groups. Therefore, how many groups contain mutants? This also begs the question, how many mutant individuals invade each group? Moreover, in reality, there is a high possibility that the groups (herds) involve individuals that are blood relatives. Then, an invasion by mutant individuals does not mean that a very small number of mutant individuals enters a great number of groups and this generates a great number of mutant groups. It may mean that a very small number of groups turn into mutant groups. Therefore, if the number of existing mutant groups is  $p$ , it is certain that  $p < \epsilon N$ . In the inter-group game, however, note that the number of all groups (players) may vary according to how big  $n$  is. Let  $\mu$  be the share of mutant groups in all groups, then,

$$\mu = p / (N/n) < (\epsilon N) / (N/n) = \epsilon n. \tag{2.5}$$

If  $n$  is small (not relatively, but absolutely), then  $\mu$  is small. Otherwise, the mutant groups intrude at the same time with high proportions. Under such a situation, the ESS condition is inadequate as the criterion of a stable equilibrium. Conversely, when we try to model a phenomenon, if the assumption that  $n$ , the number of individuals in the group, does not exceed a certain limit is reasonable, we can use an ESS as an adequate solution in the inter-group evolutionary game.

How shall we approach this question if we cannot consider  $n$  as small? In this case, the solution may be obtained by using a method that involves the iterated elimination of strictly dominated strategies. Of course, this is also correct when  $n$  is small. However, if this strategy is used, the scope of corresponding phenomena that can be modeled is naturally very limited.

Conversely, it is possible to weaken the concept of the solution. This is because the fact that mutant individuals intrude into a large population and occupy a certain proportion of the population suggests that the proportion of mutant individuals in a mutant group may be limited.

In this respect, it is assumed that the proportion of mutant individuals in a mutant group is a constant  $\bar{v}$ , and the other groups comprise only individuals choosing incumbent individual strategies. When equation (2.4) is simplified  $\bar{v} = \epsilon / \mu$ , by  $\epsilon N = (n \bar{v} \mu \cdot (N/n))$ . From equation (2.5),  $\mu < \epsilon n$ . Thus,  $\bar{v} > 1/n$ .

Therefore, if the other variables are given, the proportion  $\bar{v}$  of the mutant individuals in the mutant group varies according to  $n$ , the number of individuals in a group. This means that when we solve the inter-group game by determining  $n$  at a certain value, we need not consider all kinds of invasions of the mutant group. In other words, this means that ‘the ESS for limited (mutant group) strategies’ (and not for any strategy) may be regarded as the solution.

Taking this into account, let us continue the discussion by regarding an ESS as the solution of the inter-group game.

### 3. The definition of the NPD condition

A game  $G=(I, S, \pi)$  meets the “NPD condition” (the n-person prisoner’s dilemma) if

$$\forall i \in I, \exists c, d \in S_i, \forall s \in S, \pi_i(d, s_{-i}) > \pi_i(c, s_{-i}) \text{ and } \pi_i(d, (d, \dots, d)_{-i}) < \pi_i(c, (c, \dots, c)_{-i}) \quad (3.1)$$

In a game, Let  $I' \subset I$  be a subset of the player’s set and let  $s'$  be their strategy profile. The game meets the “NPD condition in  $I'$ ” if

$$\begin{aligned} &\forall i \in I', \exists c, d \in S_i, \forall s' \in S', \forall s \in S, \\ &\pi_i(d, s'_{-i}, s_{-I'}) > \pi_i(c, s'_{-i}, s_{-I'}) \text{ and} \\ &\pi_i(d, (d, \dots, d)_{-i}, s_{-I'}) < \pi_i(c, (c, \dots, c)_{-i}, s_{-I'}) \end{aligned} \quad (3.2)$$

Apparently, if a game meets the “NPD condition”, it meets the “NPD condition in  $I'$ ” for

any  $I' \subset I$ . If it is so defined, some of the phenomena that were adequately modeled as an NPD game can be re-modeled as a game that meets the “NPD condition in  $I'$ ”.

Note that an ordinary NPD game can be reconstructed in this way to become a game that meets the “NPD condition in  $I'$ ” because a new dimension has been introduced to the model. In order to understand this, we can consider reconstructing a “two-person PD game” as a “repeated two-person PD game”. This differs from changing a PD game into another game belonging to the same class, *e.g.*, into a coordination game, which belongs to the same class. This is also apparent from the fact that two-person games other than PD games can be reconstructed as repeated games. Moreover, an ordinary PD game can be repositioned as a specific example of a repeated PD game. Similarly, (3.2) represents an example of a game of a new class.

#### 4. The definition of a war game

In a game where two groups  $x$  and  $y$  exist, it is assumed that group strategies are written as  $x: (\alpha_x, 1-\alpha_x)$  and  $y: (\alpha_y, 1-\alpha_y)$ . Let  $(\alpha_i, 1-\alpha_i)$  be an individual's strategy. Then, a payoff function for the individual can be expressed as follows using real numbers  $s$ ,  $k$ , and  $t$ :

$$\begin{aligned} \pi_{i \in x}(\alpha_i, \alpha_x, \alpha_y) &= k(1-\alpha_i) + s\alpha_x + t\alpha_y, \\ \pi_{i \in y}(\alpha_i, \alpha_y, \alpha_x) &= k(1-\alpha_i) + s\alpha_y + t\alpha_x, \quad (\text{where } s > k > s + t > 0). \end{aligned} \quad (4.3)$$

The condition  $s > k > s + t$  can be estimated from a specific empirical fact when the individual strategy is “cooperation or non-cooperation in the war”. Conversely, because the payoff in an evolutionary game is fitness, the condition  $s + t > 0$  is derived from the general fact that fitness may be lower than 1, but is never lower than 0.

For an individual in group  $x$ , if  $\alpha_y$  is given, the best response of each individual  $i$  is

determined by  $\alpha_i=0$  whatever  $\alpha_x$  may be. By contrast,  $\pi_i \in_x(0,0,\alpha_y) = k + t\alpha_y < s + t\alpha_y = \pi_i \in_x(1,1,\alpha_y)$ . Naturally, the same applies to the individuals in group  $y$ . Therefore, the war game meets the NPD condition in both groups  $x$  and  $y$ .

Furthermore, the following points are very characteristic of this game. As seen above, from the perspective of an individual in group  $x$ , the “rational choice” of group  $x$  is  $\alpha_x=1$ , whatever the group strategy of group  $y$  may be. That is,  $\forall i \in x, \forall \alpha_y, \pi_i \in_x(\alpha_i, 1, \alpha_y) = k(1-\alpha_i) + s + t\alpha_y \geq k(1-\alpha_i) + s\alpha_x + t\alpha_y = \pi_i \in_x(\alpha_i, \alpha_x, \alpha_y)$ . Similarly, if seen from the perspective of an individual in group  $y$ , the “rational choice” of group  $y$  is  $\alpha_y=1$ . By contrast, however,  $\pi_i(1,1,1) = s + t < k = \pi_i(0,0,0)$ . Therefore, this gives rise to another Prisoners’ Dilemma.

One difficulty in solving the dilemma between the groups is that they may apply strong pressure on the choices of the individuals that constitute each of these groups. As a result, we can solve a Prisoners’ Dilemma between the individuals inside the group.

## 5. The solution of the NPD

*Theorem* In a war game, the group strategy  $(0,1)$  comprising the individual strategy  $(0,1)$  is not an ESS.

*Proof* It is assumed that, in the profile of a war game, two types of individual strategies are present, *i.e.*, an incumbent strategy  $x: (x, 1-x)$  and a mutant individual strategy  $w: (w, 1-w)$  ( $1 \geq w > 0$ ). Simultaneously, two types of group strategies are present, *i.e.*, an incumbent strategy  $x: (x, 1-x)$  and a mutant group strategy  $y: (y, 1-y)$ . Then, the ratio of individuals that follow individual strategy  $w$  in the group is  $0$  in the group using group strategy  $x$ , and is  $v$  ( $v \neq 0$ ) in the group using group strategy  $y$ . Therefore, when the numbers of individuals in the groups are sufficiently high,

$$y=(1-v)x + vw.$$

Incidentally, from (1,2)

$$(u_l^{(n)}(x^{(n)}), u_m^{(n)}(x^{(n)})) \begin{pmatrix} v_{l,xk} \\ \\ \\ v_{m,xk} \end{pmatrix} = v_{l,xk} \cdot u_{i \in \{l\}}(x_{i,xk}, x_{l-i}^{(n)}, x_m^{(n)}) + v_{m,xk} \cdot u_{j \in \{m\}}(x_{j,xk}, x_l^{(n)}, x_{m-j}^{(n)}) \quad (5.1)$$

Therefore, in an interaction between the group using group strategy  $x$  and the group using group strategy  $y$ , if  $x_k=w$ , then

$$(u^{(n)}(x,y), u^{(n)}(y,x)) \begin{pmatrix} 0 \\ \\ \\ v \end{pmatrix} = v \cdot u(w,y,x) \quad (5.2)$$

Therefore,  $u^{(n)}(y,x)=u(w,y,x)$

If  $x_k=x$  in an interaction between the groups using group strategy  $x$ ,

$$(u^{(n)}(x,x), u^{(n)}(x,x)) \begin{pmatrix} 1 \\ \\ \\ 1 \end{pmatrix} = 2u(x,x,x) \quad (5.3)$$

Therefore,  $u^{(n)}(x,x)=u(x,x,x)$

Here, it is assumed that  $Z=u^{(n)}(x,x) - u^{(n)}(y,x)$ . In the war game, if group strategy  $x$  is an ESS,

$$\begin{aligned} Z &= u^{(n)}(x,x) - u^{(n)}(y,x) \\ &= u(x,x,x) - u(w,y,x) \\ &= u(x,x,x) - u[w, (1-v)x + vw, x] \\ &= [k(1-x) + sx + tx] - [k(1-w) + s\{(1-v)x + vw\} + tx] \\ &= (sv-k)(x-w) \geq 0. \end{aligned} \quad (5.4)$$

When  $x=(0,1)$ ,  $w \succ x$  for any  $w$ . Since it is assumed that  $s > 0$ , if  $x$  is an ESS,  $v \leq k/s$  for any  $v$  that meet the condition  $0 < v \leq 1$ . That is,  $1 \leq k/s$  is the necessary condition for  $x:(0,1)$  being an ESS. However, because  $s > k$ ,  $1 > k/s$ . Hence,  $x:(0,1)$  is not an ESS. ■

Incidentally, what if  $x=(1,0)$ ? For any  $w$ , we have  $w \succ x$ . If  $x$  is an ESS,  $v \geq k/s$  for any  $v$  that meet the condition  $0 < v \leq 1$ , *i.e.*,  $k/s < 0$ . However, because  $s > k > 0$ ,  $k/s > 0$ . Hence, this strategy is also not an ESS. Then, based on the discussion in the latter half of Section 2, we try to weaken the concept of the solution. Here, it is assumed that the mutant group strategy of any  $v$  that do not always meet the condition  $0 < v \leq 1$ , but meet a condition  $0 < v \leq k/s (< 1)$  may intrude. In this case, naturally,  $(0,1)$  is an “ESS”. Similarly, when only the mutant strategy under the condition  $1 > v \geq k/s (> 0)$  may intrude,  $(1,0)$  is an “ESS”. Therefore, if it is assumed that  $v > 1/n$  in reality, when  $1/n \geq k/s$ , only  $(1,0)$  is an “ESS” for any  $v$  under the condition  $1 > v > 1/n (\geq k/s > 0)$ . By contrast, when  $1/n \leq k/s$ , neither  $(0,1)$  nor  $(1,0)$  are an “ESS”. If  $s$  and  $k$  have certain values, the smaller  $n$  is, the greater the possibility that the condition  $1/n \geq k/s$  exists. Therefore, the smaller the number of individuals that constitute the group, *i.e.*, the bigger the number of groups, the greater the possibility that  $(1,0)$  is an ESS. (In our society, however, “all Cs” are not selected under the condition of the dilemma, and it is doubtful whether this interesting discussion has realistic implications.)

Note that parameter  $t$  is not included in (5.4). This means that we can solve the NPD at any time if groups are related to each other in some interaction; if not, the interaction is a war. This indicates the high degree to which the theorem can potentially be generalized, and also, the high possibility that the NPD is solved in reality. However, it is rather important from the viewpoint of empirical science that we can at least show one example of the interactions between groups and that we can specify its payoff function from facts.

Through frequent contacts between groups in the past, we can estimate whether there was a genetic force capable of suppressing rational choices in NPD situations. Naturally, no group (herd) exists at present in a literal sense. However, it exists in a cognitive framework, by which we recognize our circumstances. This “Residue” (if we use the term of V. Pareto) influences our behavior. When we find a “group”, to which we ourselves belong, and when we find another “group” related to such a group, we cherish a certain type of emotion, which competes with rational thinking, and we adopt behaviors that counter our simple theoretical prediction.

Hideki Kamiyama rxg00156@nifty.ne.jp  
(Japan Society for the Promotion of Science)  
XV World Congress of Sociology 11/7/2002



## References

- Axelrod, Robert, 1984, *The evolution of cooperation*, New York : Basic Books.
- Coleman, James S. , 1990, *Foundations of social theory*, Cambridge, Mass. : Belknap Press of Harvard University Press.
- Hardin, Garrett, 1968, "The Tragedy of the Commons," *Science*162:1243-1248.
- Hardin, Russell, 1971, "Collective Action as an Agreeable n-Prisoners' Dilemma," *Behavioral Science*,16:472-481.
- Hobbes, Thomas, 1651=1965, *Leviathan*, Oxford:Clarendon Press.
- Olson, Mancur, 1965, *The logic of collective action: public goods and the theory of groups*, Cambridge: Harvard University Press.
- Ooura, Hirokuni, 2002, "Evolution of Agriculture Strategy –Dilemma Avoidance by Non-random Model–," (A Paper for Mathematical Sociology in Japan and America May31-June2, 2002, Vancouver, Canada).
- 
- Parsons, Talcott, 1937, *The structure of social action*, MacGraw-Hill.
- Runciman, W.G. and A. Sen, 1965, "Games, Justice and the General Will," *Mind*74:554-62.
- Taylor, Michael, 1987, *The possibility of cooperation*, New York : Cambridge University Press.
- Ullman-Margalit, Edna, 1977, *The emergence of norms*, Oxford : Clarendon Press.
- Weibull, Jorgen W., 1995, *Evolutionary game theory*, Cambridge, Mass. : MIT Press.
- Wilson, David S., 1975, "A theory of Group Selection," *Proceedings of the National Academy of Science of the United States of America*,72:143-146.

***This report is a part of result by the subsidy for the science research from the Ministry of Education, Culture, Sports, Science and Technology of Japan.***